**10/21/2022 – Friday - How to Analyze, Design and Model a Quadcopter Airframe – Part 1**

This purpose of this paper is to teach you how to analyze, design, and model a quadcopter airframe. A spreadsheet is used to initially examine the forces and torques applied to the airframe. This information will provide an analysis method used to design and size the quadcopter, and to provide a six-degree-of-freedom dynamic simulation model of its airframe, motors, and propellers. The control forces and torques are examined to determine if they are great enough to balance the quadcopter airframe to determine if the design is feasible. The controls laws for the quadcopter will be addressed at a later time. It is assumed that each of the 4 motors will be commanded by providing an RPM command which then each motor will turn each propeller at the commanded RPM. Examples calculations for this example are performed at sea-level to provide one design point that is used to verify the calculations in the spreadsheet. Parametric analysis can then be performed to optimize the design.

**Top View Diagram of Quadcopter Looking Down:**



**Side View Diagram of Quadcopter – In Hover:**



**Side View Diagram of Quadcopter – Pitch Nose down for Forward Flight:**



**Body Axis Moments:**

**Assume Thin Disk for Mass Moment of Inertia:**

**Propeller Mass Moment of Inertia Assume Slender Rod:**

**Example Calculations in this example:**

Arm Radius of quadcopter:

Arm Angle of quadcopter:

Crank Arm:

Crank Arm:

Mass of the quadcopter:

X & Y - Mass Moment of Inertia of the quadcopter:

Z – Mass Moment of Inertia of the quadcopter:

Mass of the Propeller:

Radius of the Propeller:

Mass Moment of Inertia of the Propeller:

In this example, we have four two bladed 6” x 3 propellers for the quadcopter that are spinning at a RPM that is sufficient to be able to lift the weight of the quadcopter. This RPM will be computed in this example. The chord of the blade is 0.6 inches.

Determine Blade Pitch:

D = 6 inches = Diameter

P = 3 = Pitch Displacement = for every revolution of the propeller, 3 inches of air is displaced.

C = 0.6 inches = Blade Chord

R = D/2 = 3 inches = Radius of the propeller

Propeller Diameter:

Propeller Radius:

Propeller Inflow:

Propeller Chord:

Propeller Surface Area:

Propeller Pitch:

Density of air at sea level:

Coefficient of Lift for a Flat Plate:

Coefficient of Drag for an Airfoil:

Coefficient of Drag for a Flat Plate:

Where Pc is the chord of the propeller, the surface area of the propeller is:

Where x = A, B, C, & D, qave for each propeller is:

Where x = A, B, C, & D, Fx for each propeller blade is:

For a 2-bladed propeller, the thrust is:

The force gain of the propeller as a function of its rotational speed squared is:

The motor speed required to lift the weight of the quadcopter can be determined by the following equation:

D1 Drag for the Propeller Blade is:

D2 Drag for the Propeller Blade is:

Total Drag for a two bladed Propeller is:

Torque on the propeller is:

Power required for the propeller is:

**Single Propeller Sample Calculations – Assume all four motors have the same command – Test Case**

Propeller force Gain:

Motor speed required for the 4 motors to lift the weight of the quadcopter:

Single Propeller Thrust: (one quarter of the quadcopter weight):

D1 drag for a single propeller:

D2 drag for a single propeller:

Total drag for a single propeller:

Torque for a single propeller:

Power for a single propeller:

Power for all 4 propellers:

Battery Voltage:

Battery Energy:

Battery Life:

**Multiple Propeller Commands – Where the commands are different between the four motors - Test Case:**

Let’s say the motors are commanded to run at a certain RPM.

Each Motor A, B, C & D will get a RPM command to maneuver the aircraft.

Let:

**Motor Angular Velocities:**

**Dynamic Pressure of Each Propeller:**

**Thrust of each propeller:**

**Roll Moments:**

**Pitch Moment:**

**Drag1 for Each Propeller Blade:**

**Drag2 for Each Propeller Blade:**

**Total Drag for Each Propeller:**

**Torque for Each Propeller:**

**Yaw Moment:**

**Power for Each Propeller:**

**Total Power for all Four Propellers:**

**Computing Airframe Drag:**

Width of Airframe:

Length of Airframe:

Height of Airframe:

Estimated Pitch Attitude:

Estimated Roll Attitude:

X-Surface Area:

Y-Surface Area:

Forward Airspeed (Vx):

Lateral Airspeed (Vy):

Forward Dynamic Pressure (qx):

Lateral Dynamic Pressure (qy):

D1 Longitudinal Drag:

D1 Lateral Drag:

D2 Longitudinal Drag:

D2 Lateral Drag:

D Total Longitudinal Drag:

D Total Lateral Drag:

The Pitch Attitude is estimated to be 10 degrees.

The Roll Attitude is estimated to be 10 degrees.

Xt – Estimated Longitudinal Force from Props:

Yt – Estimated Lateral Force from Props:

Xr – Total Residual X Force (Slight Forward Force):

Yr – Total Residual Y Force (Slight Right Force):

Zr – Total Residual Z Force (Slight Upward Force):

**Equations of Motion for the Six Degree of Freedom aircraft model:**

Six-Degrees-of-Freedom - Equations of Motion for an air vehicle on earth

Note: since this term angular velocities always sums to zero, the gyroscopic torques can be ignored to simplify the equations of motion.

Note: since this term angular accelerations always sums to zero, the counter torques can be ignored to simplify the equations of motion. Also since then the term can be ignored to simplify the equations of motion.

Body Axis Forces

Body Axis Angular Rates to Euler Angle Rates

Body Axis Linear Velocities to Earth Axis Linear Velocities