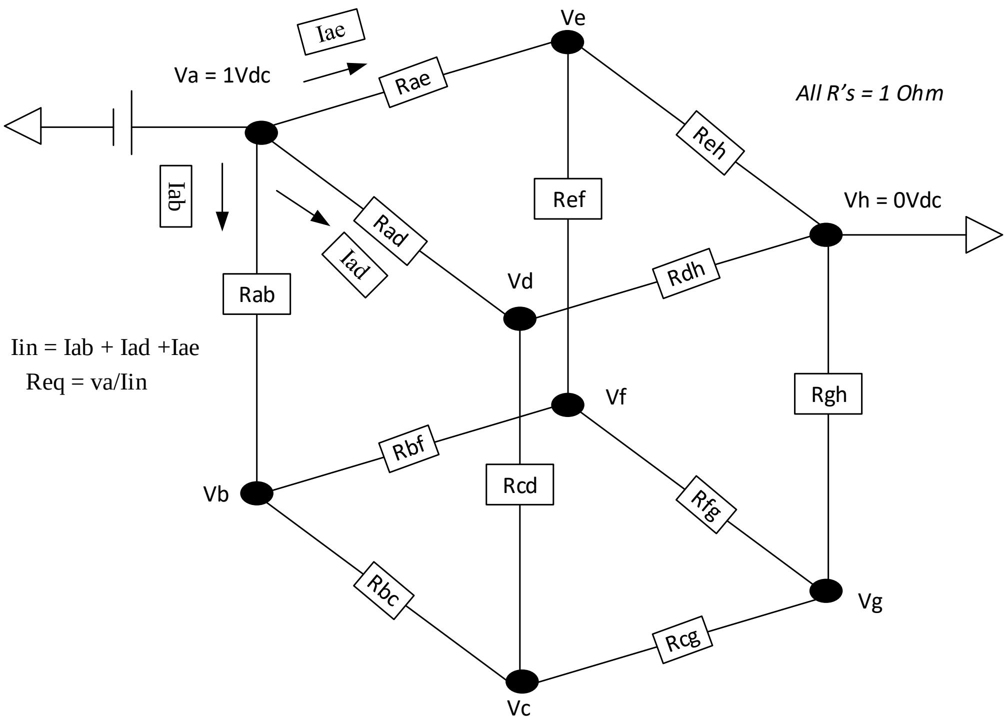
07/20/2022 – Wednesday – Resistance Cube Puzzle – A More General Solution Method.

I would like to first give you the answers to the problem, then show you how to perform the analysis to determine the answer. There are three different equivalent resistances on this cube that can be determined. Thereby, three examples are provided in this lecture.

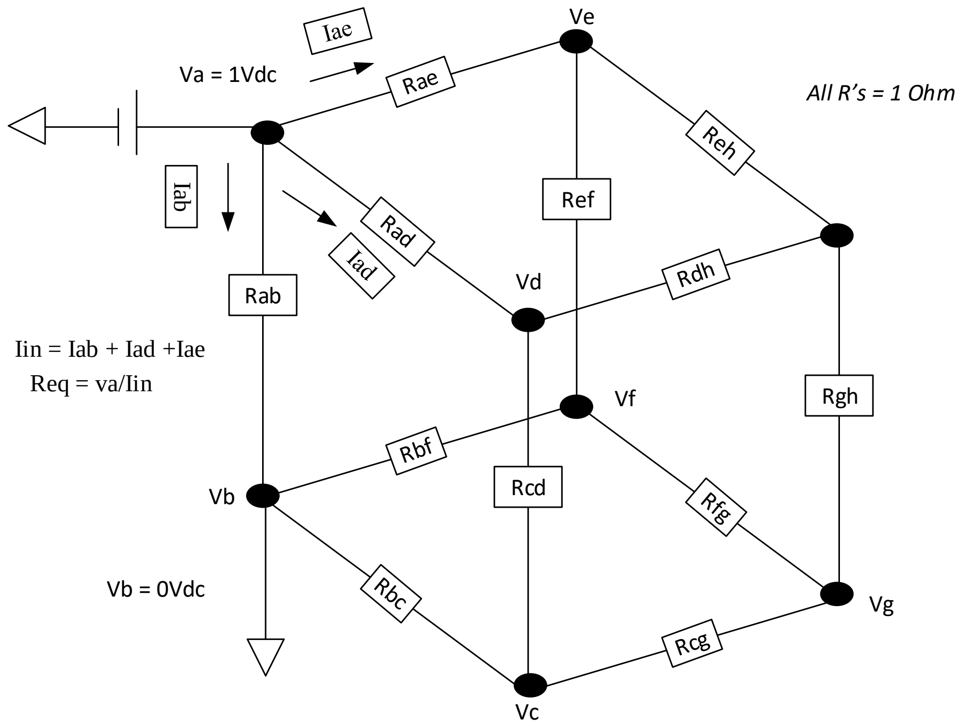
Example 1 – Diagonal Equivalent Resistance: (Ohm Meter Reads between points "a" and "h".)

Example 1
Question:
If all R's = R, determine the equivalent resistance from point a to point h (Req_ah)?
Answer:
Req_ah = 3R/4



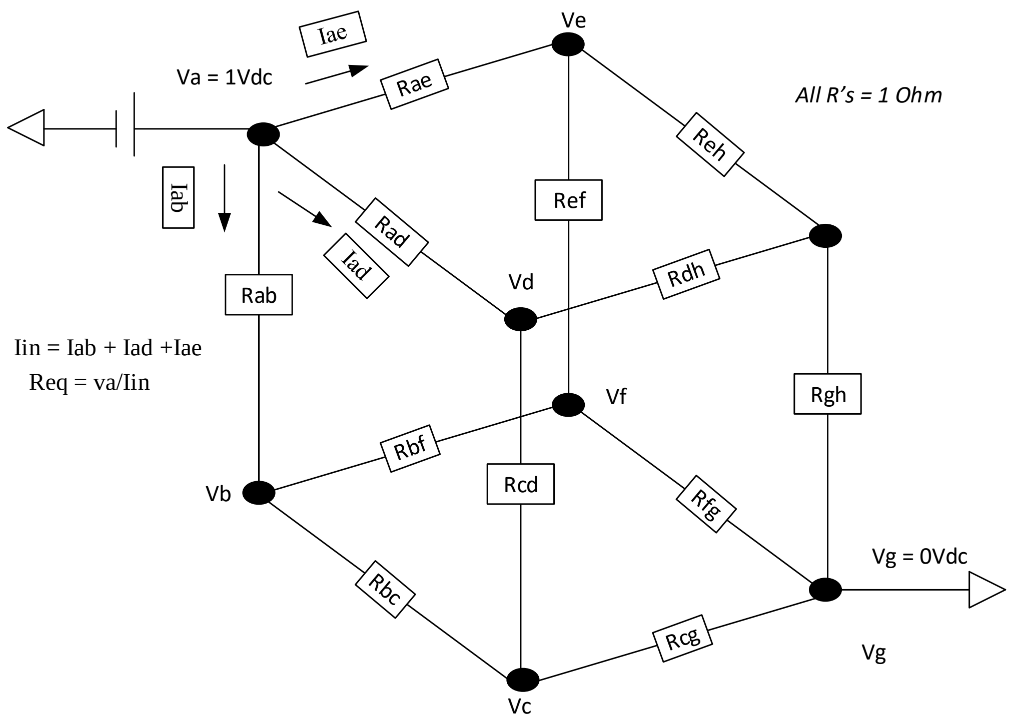
Example 2 – Edge Equivalent Resistance: (Ohm Meter Reads between points "a" and "b".)

Example 2
Question:
If all R's = R, determine the equivalent resistance from point a to point b (Req_ab)?
Answer:
Req_ab = 7R/12



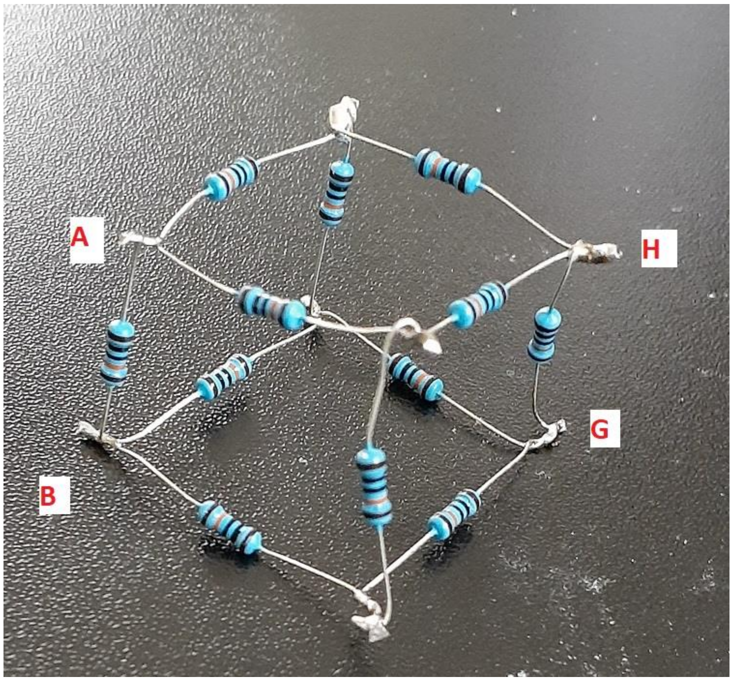
Example 3 – Longest Corner to Corner Equivalent Resistance: (Ohm Meter Reads between points "a" and "g".)

Example 3
Question:
If all R's = R, determine the equivalent resistance from point a to point g (Req_ag)?
Answer:
Req_ag = 5R/6



Measured Values

Using Twelve (12) 100K resistors, an experimental cube was built to be measured.



Resistance from point A to Point H was measured to be: = 74.6K, Expected Value = 3R/4 = 75.0K.
Resistance from point A to Point B was measured to be: = 57.9K, Expected Value = 7R/12 = 58.3K.
Resistance from point A to Point G was measured to be: = 83.0K, Expected Value = 5R/6 = 83.3K.

There have been many on-line videos that show you how to solve this problem. However, they are very limited, and use too many assumptions to simplify the circuit. Today, I am going to show you a new technique to solve this type of problem. This technique will provide the following benefits over many of the other methods shown. As an electrical engineer, I have been using this technique for many years and have solve many hard electronics issues using this method that I am going to teach you. So far I haven't found any engineering text-books that will show you this method. With this technique you can solve most circuits. Learning this method will make you a much better capable electrical engineer.

Here are some of the advantages of this method and procedure:

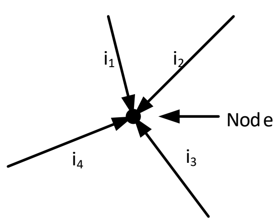
1) You don't have to redraw the circuit to two dimensions.
2) You don't have to do any creative thinking or make any assumptions when using this method.
3) You don't have to worry about symmetry to simplify the circuit.
4) SPICE is not used or needed.
5) You won't have a high work-load in having to solve equations by hand.
6) It will find a solution whether all of the 12 resistors are the same or different.
7) It will find a solution for all other geometric shapes such as a tetrahedron, octahedron, etc.
8) It will work if dynamic passive components such as capacitors or inductors are introduced. Thereby, the equivalent impedance or transfer functions are determined rather than the equivalent resistance.
9) It can also be used to determine all of the voltages and currents in the circuit.

Here is the general procedure that we will go through in this presentation:

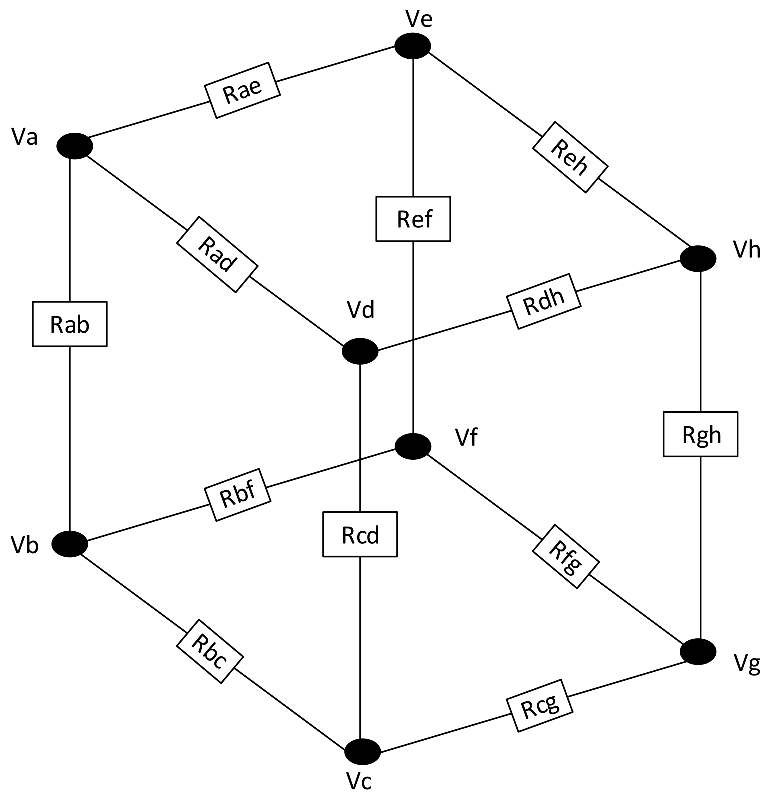
1) Write the Nodal Equations.
2) Use Public Domain Application "Maxima" to solve the equations. (Note, they could be solved by hand, but that will cause a very high work-load, and be labor intensive, and error prone.)
3) Determine the Drive Input Current.
4) Assume the input voltage is 1 Vdc.
5) Determine the equivalent resistance from steps 3 & 4. Req = Vin/Iin (Thevenin's Theorem)

How to write the Nodal Equations?

For each node, assume all currents enter the node as shown in the following figure:
The sum of these currents going into the node is zero.

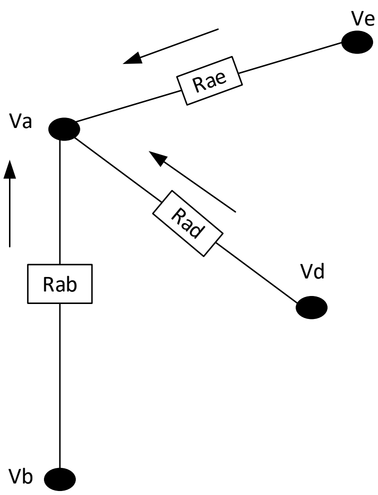


Here is a way to solve the resistance cube puzzle.



Write the Nodal Equations:
There are 12 Resistors in the Cube Problem.
There are eight nodes in the cube.
There are eight equations and eight unknowns.

Nodal Equation for node "a".



Equation 1 (a)

$$\frac{(V_a - V_e)}{R_{ae}} + \frac{(V_a - V_d)}{R_{ad}} + \frac{(V_a - V_b)}{R_{ab}} = 0$$

Note that for the cube there is a pattern where there will be three terms in the equation. Each term begins with "Va" as shown.

Note that Rad = Rda.

Here are the eight equations and eight unknowns.

Equation 1 (node a)

$$\frac{(V_a - V_e)}{R_{ae}} + \frac{(V_a - V_d)}{R_{ad}} + \frac{(V_a - V_b)}{R_{ab}} = 0$$

Equation 2 (node b)

$$\frac{(V_b - V_a)}{R_{ab}} + \frac{(V_b - V_f)}{R_{bf}} + \frac{(V_b - V_c)}{R_{bc}} = 0$$

Equation 3 (node c)

$$\frac{(V_c - V_b)}{R_{bc}} + \frac{(V_c - V_g)}{R_{cg}} + \frac{(V_c - V_d)}{R_{cd}} = 0$$

Equation 4 (node d)

$$\frac{(V_d - V_a)}{R_{ad}} + \frac{(V_d - V_h)}{R_{dh}} + \frac{(V_d - V_c)}{R_{cd}} = 0$$

Equation 5 (node e)

$$\frac{(V_e - V_f)}{R_{ef}} + \frac{(V_e - V_a)}{R_{ae}} + \frac{(V_e - V_h)}{R_{eh}} = 0$$

Equation 6 (node f)

$$\frac{(V_f - V_e)}{R_{ef}} + \frac{(V_f - V_g)}{R_{fg}} + \frac{(V_f - V_b)}{R_{bf}} = 0$$

Equation 7 (node g)

$$\frac{(V_g - V_f)}{R_{fg}} + \frac{(V_g - V_h)}{R_{gh}} + \frac{(V_g - V_c)}{R_{cg}} = 0$$

Equation 8 (node h)

$$\frac{(V_h - V_d)}{R_{dh}} + \frac{(V_h - V_e)}{R_{eh}} + \frac{(V_h - V_g)}{R_{gh}} = 0$$

Put the equations in Maxima to solve them:

```
eq1:(va-ve)/Rae+(va-vd)/Rad+(va-vb)/Rab;
eq2:(vb-va)/Rab+(vb-vf)/Rbf+(vb-vc)/Rbc;
eq3:(vc-vb)/Rbc+(vc-vg)/Rcg+(vc-vd)/Rcd;
eq4:(vd-va)/Rad+(vd-vh)/Rdh+(vd-vc)/Rcd;
eq5:(ve-vf)/Ref+(ve-va)/Rae+(ve-vh)/Reh;
eq6:(vf-ve)/Ref+(vf-vg)/Rfg+(vf-vb)/Rbf;
eq7:(vg-vf)/Rfg+(vg-vh)/Rgh+(vg-vc)/Rcg;
eq8:(vh-vd)/Rdh+(vh-ve)/Reh+(vh-vg)/Rgh;
eq9:solve([eq1,eq2,eq3,eq4,eq5,eq6,eq7,eq8],[va,vb,vc,vd,ve,vf,vg,vh]);
```

Results:

```
(%i9) eq1:(va-ve)/Rae+(va-vd)/Rad+(va-vb)/Rab;
      eq2:(vb-va)/Rab+(vb-vf)/Rbf+(vb-vc)/Rbc;
      eq3:(vc-vb)/Rbc+(vc-vg)/Rcg+(vc-vd)/Rcd;
      eq4:(vd-va)/Rad+(vd-vh)/Rdh+(vd-vc)/Rcd;
      eq5:(ve-vf)/Ref+(ve-va)/Rae+(ve-vh)/Reh;
      eq6:(vf-ve)/Ref+(vf-vg)/Rfg+(vf-vb)/Rbf;
      eq7:(vg-vf)/Rfg+(vg-vh)/Rgh+(vg-vc)/Rcg;
      eq8:(vh-vd)/Rdh+(vh-ve)/Reh+(vh-vg)/Rgh;
      eq9:solve([eq1,eq2,eq3,eq4,eq5,eq6,eq7,eq8],[va,vb,vc,vd,ve,vf,vg,vh]);
```

(eq1) $$\frac{va-ve}{Rae}+\frac{va-vd}{Rad}+\frac{va-vb}{Rab}$$

(eq2) $$\frac{vb-vf}{Rbf}+\frac{vb-vc}{Rbc}+\frac{vb-va}{Rab}$$

(eq3) $$\frac{vc-vg}{Rcg}+\frac{vc-vd}{Rcd}+\frac{vc-vb}{Rbc}$$

(eq4) $$\frac{vd-vh}{Rdh}+\frac{vd-vc}{Rcd}+\frac{vd-va}{Rad}$$

(eq5) $$\frac{ve-vh}{Reh}+\frac{ve-vf}{Ref}+\frac{ve-va}{Rae}$$

(eq6) $$\frac{vf-vg}{Rfg}+\frac{vf-ve}{Ref}+\frac{vf-vb}{Rbf}$$

(eq7) $$\frac{vg-vh}{Rgh}+\frac{vg-vf}{Rfg}+\frac{vg-vc}{Rcg}$$

(eq8) $$\frac{vh-vg}{Rgh}+\frac{vh-ve}{Reh}+\frac{vh-vd}{Rdh}$$

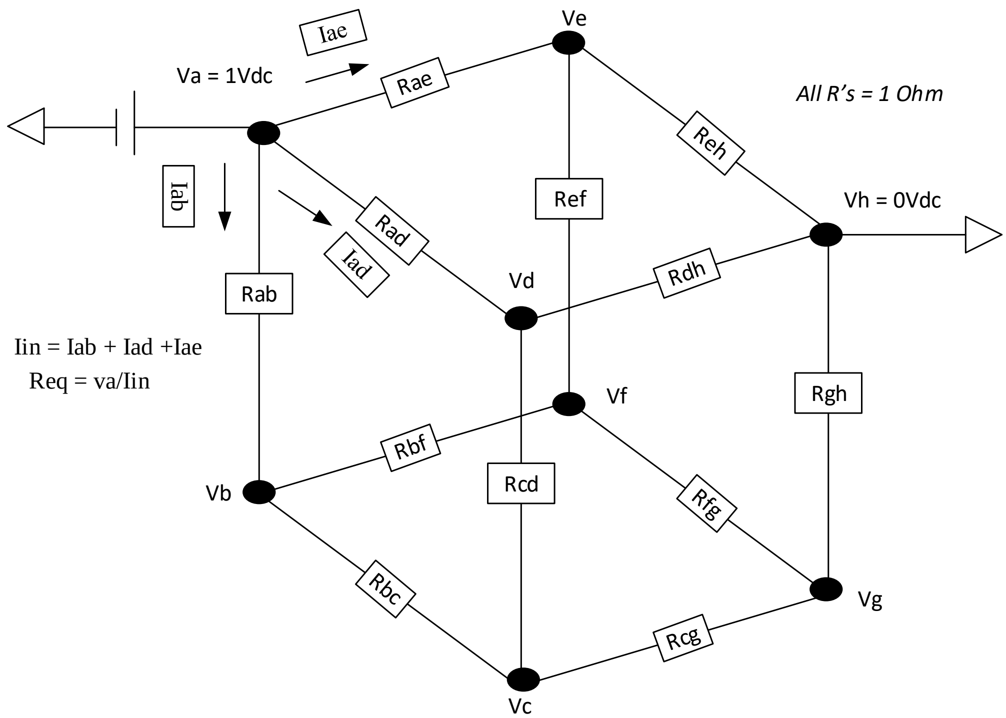
solve: dependent equations eliminated: (6)

(eq9) $$[[va=\%r1, vb=\%r1, vc=\%r1, vd=\%r1, ve=\%r1, vf=\%r1, vg=\%r1, vh=\%r1]]$$

Note that Maxima can't solve the equations because they are linearly dependent (due to symmetry). So some of the equation need to be eliminated.

Let's try example 1 with all different "R's":

Example 1
Question:
If all R's = R, determine the equivalent resistance from point a to point h (Req_ah)?
Answer:
Req_ah = 3R/4



Note that in example 1, the Ohmmeter is placed between points "a" and "h".
So, "Va" is known and assume it is 1 Vdc. "Vh" is known and assume it is 0 Vdc.

Since "Va" and "Vh" are known, the technique is to remove nodal "equation 1" for "Va" and nodal "equation 8" for "Vh".

Input to Maxima:

```
eq1:(va-ve)/Rae+(va-vd)/Rad+(va-vb)/Rab;
eq2:(vb-va)/Rab+(vb-vf)/Rbf+(vb-vc)/Rbc;
eq3:(vc-vb)/Rbc+(vc-vg)/Rcg+(vc-vd)/Rcd;
eq4:(vd-va)/Rad+(vd-vh)/Rdh+(vd-vc)/Rcd;
eq5:(ve-vf)/Ref+(ve-va)/Rae+(ve-vh)/Reh;
eq6:(vf-ve)/Ref+(vf-vg)/Rfg+(vf-vb)/Rbf;
eq7:(vg-vf)/Rfg+(vg-vh)/Rgh+(vg-vc)/Rcg;
eq8:(vh-vd)/Rdh+(vh-ve)/Reh+(vh-vg)/Rgh;
eq9:solve([eq2,eq3,eq4,eq5,eq6,eq7],[vb,vc,vd,ve,vf,vg]),va=1,vh=0;
```

(%i9)

```
eq1:(va−ve)/Rae+(va−vd)/Rad+(va−vb)/Rab;
eq2:(vb−va)/Rab+(vb−vf)/Rbf+(vb−vc)/Rbc;
eq3:(vc−vb)/Rbc+(vc−vg)/Rcg+(vc−vd)/Rcd;
eq4:(vd−va)/Rad+(vd−vh)/Rdh+(vd−vc)/Rcd;
eq5:(ve−vf)/Ref+(ve−va)/Rae+(ve−vh)/Reh;
eq6:(vf−ve)/Ref+(vf−vg)/Rfg+(vf−vb)/Rbf;
eq7:(vg−vf)/Rfg+(vg−vh)/Rgh+(vg−vc)/Rcg;
eq8:(vh−vd)/Rdh+(vh−ve)/Reh+(vh−vg)/Rgh;
eq9:solve([eq2,eq3,eq4,eq5,eq6,eq7],[vb,vc,vd,ve,vf,vg]),va=1,vh=0;
```

(eq1) $$\frac{va-ve}{Rae}+\frac{va-vd}{Rad}+\frac{va-vb}{Rab}$$

(eq2) $$\frac{vb-vf}{Rbf}+\frac{vb-vc}{Rbc}+\frac{vb-va}{Rab}$$

(eq3) $$\frac{vc-vg}{Rcg}+\frac{vc-vd}{Rcd}+\frac{vc-vb}{Rbc}$$

(eq4) $$\frac{vd-vh}{Rdh}+\frac{vd-vc}{Rcd}+\frac{vd-va}{Rad}$$

(eq5) $$\frac{ve-vh}{Reh}+\frac{ve-vf}{Ref}+\frac{ve-va}{Rae}$$

(eq6) $$\frac{vf-vg}{Rfg}+\frac{vf-ve}{Ref}+\frac{vf-vb}{Rbf}$$

(eq7) $$\frac{vg-vh}{Rgh}+\frac{vg-vf}{Rfg}+\frac{vg-vc}{Rcg}$$

(eq8) $$\frac{vh-vg}{Rgh}+\frac{vh-ve}{Reh}+\frac{vh-vd}{Rdh}$$

(eq9) [[vb=(Rbf (Rbc (Rcg
(Rdh (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rcd
(Rdh (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rdh
(Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh)+Rad
(Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh))+Rcg (Rcd
(Rdh (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rad Rdh
(Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rab (Rdh
(Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rad Reh Rgh+Rae Ref Rgh)+Rcg Rdh
(Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rcd (Rdh Reh Rgh+Rad Reh Rgh))+Rcd
(Rdh (Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh)+Rad (Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh))
+Rad Rdh (Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh))+Rab (Rcg
(Rdh (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rad Reh (Rgh+Rfg)+Rae Ref (Rgh+Rfg))+Rcd (Rdh Reh (Rgh+Rfg)+Rad Reh (Rgh+Rfg)))
+Rdh (Reh (Ref Rfg Rgh+Rae Rfg Rgh)+Rad Reh Rfg Rgh+Rae Ref Rfg Rgh)+Rcd (Rdh Reh Rfg Rgh+Rad Reh Rfg Rgh))+Rbc (Rcg
(Rdh (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg))+Rad (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg)))+
Rcd
(Rdh (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg))+Rad (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg)))+

(Rdh (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg))+Rad (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg)))+
Rdh (Rad (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg))+Reh (Ref Rfg Rgh+Rae Rfg Rgh)+Rae Ref Rfg Rgh)+Rab (Rdh
(Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rad Reh (Rgh+Rfg)+Rae Ref Rgh)+Rcg (Rdh Reh (Rgh+Rfg)+Rad Reh (Rgh+Rfg))+Rcd
(Rdh Reh (Rgh+Rfg)+Rad Reh (Rgh+Rfg))+Rad Reh Rfg Rgh)+Rad (Reh (Ref Rfg Rgh+Rae Rfg Rgh)+Rae Ref Rfg Rgh))+Rcg (Rcd
(Rdh (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg))+Rad (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg)))+
Rad Rdh (Reh (Ref (Rgh+Rfg)+Rae (Rgh+Rfg))+Rae Ref (Rgh+Rfg)))+Rcd
(Rdh (Reh (Ref Rfg Rgh+Rae Rfg Rgh)+Rae Ref Rfg Rgh)+Rad (Reh (Ref Rfg Rgh+Rae Rfg Rgh)+Rae Ref Rfg Rgh))+Rad Rdh
(Reh (Ref Rfg Rgh+Rae Rfg Rgh)+Rae Ref Rfg Rgh))/(Rbf (Rbc (Rcg
(Rdh (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rcd
(Rdh (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rdh
(Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh)+Rad
(Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh))+Rab (Rcg
(Rdh (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rcd
(Rdh (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rdh
(Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh)+Rad
(Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh))+Rcg (Rcd
(Rdh (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref)+Rad (Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rad Rdh
(Reh (Rgh+Rfg+Ref+Rae)+Rae (Rgh+Rfg)+Rae Ref))+Rcd
(Rdh (Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh)+Rad (Reh (Rfg Rgh+Ref Rgh+Rae Rgh)+Rae Rfg Rgh+Rae Ref Rgh))

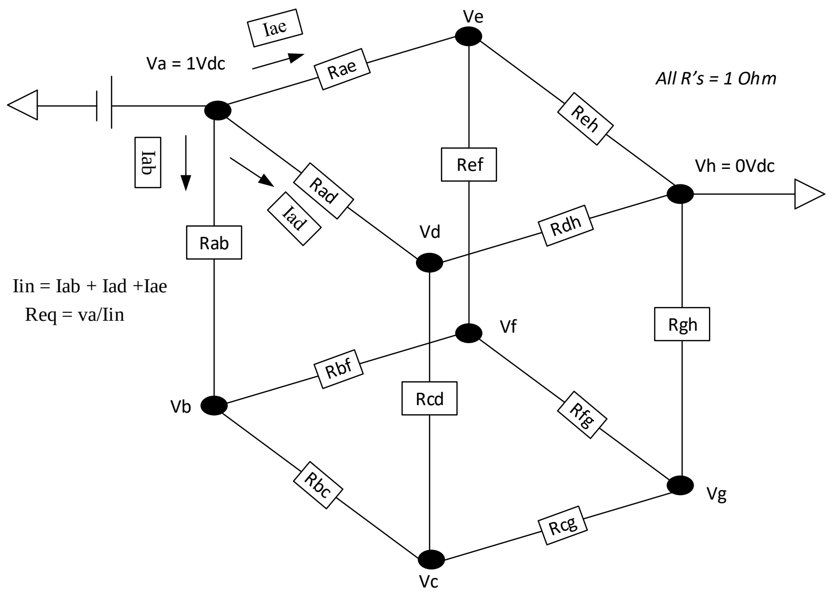
$$
\begin{aligned}
&(Rdh\,(Reh\,(Rfg\,Rgh+Ref\,Rgh+Rae\,Rgh)+Rae\,Rfg\,Rgh+Rae\,Ref\,Rgh)+Rad\,(Reh\,(Rfg\,Rgh+Ref\,Rgh+Rae\,Rgh)+Rae\,Rfg\,Rgh+Rae\,Ref\,Rgh))\\
&+Rad\,Rdh\,(Reh\,(Rfg\,Rgh+Ref\,Rgh+Rae\,Rgh)+Rae\,Rfg\,Rgh+Rae\,Ref\,Rgh))+Rbc\,(Rab\,(Rcg\\
&(Rdh\,(Reh\,(Rgh+Rfg+Ref+Rae)+Rae\,(Rgh+Rfg)+Rae\,Ref)+Rad\,(Reh\,(Rgh+Rfg+Ref+Rae)+Rae\,(Rgh+Rfg)+Rae\,Ref))+Rcd\\
&(Rdh\,(Reh\,(Rgh+Rfg+Ref+Rae)+Rae\,(Rgh+Rfg)+Rae\,Ref)+Rad\,(Reh\,(Rgh+Rfg+Ref+Rae)+Rae\,(Rgh+Rfg)+Rae\,Ref))+Rdh\\
&(Rad\,(Reh\,(Rgh+Rfg+Ref+Rae)+Rae\,(Rgh+Rfg)+Rae\,Ref)+Reh\,(Rfg\,Rgh+Ref\,Rgh+Rae\,Rgh)+Rae\,Rfg\,Rgh+Rae\,Ref\,Rgh)+Rad\\
&(Reh\,(Rfg\,Rgh+Ref\,Rgh+Rae\,Rgh)+Rae\,Rfg\,Rgh+Rae\,Ref\,Rgh))+Rcg\\
&(Rdh\,(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg))+Rad\,(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg)))+\\
&Rcd\\
&(Rdh\,(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg))+Rad\,(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg)))+\\
&Rdh\,(Rad\,(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg))+Reh\,(Ref\,Rfg\,Rgh+Rae\,Rfg\,Rgh)+Rae\,Ref\,Rfg\,Rgh)+Rad\\
&(Reh\,(Ref\,Rfg\,Rgh+Rae\,Rfg\,Rgh)+Rae\,Ref\,Rfg\,Rgh))+Rab\,(Rcg\,(Rcd\\
&(Rdh\,(Reh\,(Rgh+Rfg+Ref+Rae)+Rae\,(Rgh+Rfg)+Rae\,Ref)+Rad\,(Reh\,(Rgh+Rfg+Ref+Rae)+Rae\,(Rgh+Rfg)+Rae\,Ref))+Rdh\\
&(Rad\,(Reh\,(Rgh+Rfg+Ref+Rae)+Rae\,(Rgh+Rfg)+Rae\,Ref)+Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg))+Rad\\
&(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg)))+Rcd\\
&(Rdh\,(Reh\,(Rfg\,Rgh+Ref\,Rfg+Rae\,Rfg)+Rae\,Rfg\,Rgh+Rae\,Ref\,Rfg)+Rad\,(Reh\,(Rfg\,Rgh+Ref\,Rfg+Rae\,Rfg)+Rae\,Rfg\,Rgh+Rae\,Ref\,Rfg))+\\
&Rdh\,(Rad\,(Reh\,(Rfg\,Rgh+Ref\,Rfg+Rae\,Rfg)+Rae\,Rfg\,Rgh+Rae\,Ref\,Rfg)+Reh\,(Ref\,Rfg\,Rgh+Rae\,Rfg\,Rgh)+Rae\,Ref\,Rfg\,Rgh)+Rad\\
&(Reh\,(Ref\,Rfg\,Rgh+Rae\,Rfg\,Rgh)+Rae\,Ref\,Rfg\,Rgh))+Rcg\,(Rcd\\
&(Rdh\,(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg))+Rad\,(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg)))+\\
&Rad\,Rdh\,(Reh\,(Ref\,(Rgh+Rfg)+Rae\,(Rgh+Rfg))+Rae\,Ref\,(Rgh+Rfg)))+Rcd
\end{aligned}
$$

And many more pages.

However, it leads to very large equations when the “R’s” are different. This case can be still be easily solved by Maxima. However, we will set all of the “R’s” to be the same to simplify it.

Let's try example 1 with all "R's" the same:

> ***Example 1***
> ***Question:***
> ***If all R's = R, determine the equivalent resistance from point a to point h (Req_ah)?***
> ***Answer:***
> ***Req_ah = 3R/4***



Note that in example 1, the Ohmmeter is placed between points "a" and "h".
So, "Va" is known and assume it is 1 Vdc. "Vh" is known and assume it is 0 Vdc.

Since "Va" and "Vh" are known, the technique is to remove nodal "equation 1" for "Va" and nodal "equation 8" for "Vh".

Maxima input:

```
This solves Rah.
Remove eq1 (va) & eq8 (vh) from solve.
Set va = 1Vdc, Set vh = 0Vdc.
Got Req = 3R/4.
This is correct.

eq1:(va-ve)/R+(va-vd)/R+(va-vb)/R;
eq2:(vb-va)/R+(vb-vf)/R+(vb-vc)/R;
eq3:(vc-vb)/R+(vc-vg)/R+(vc-vd)/R;
eq4:(vd-va)/R+(vd-vh)/R+(vd-vc)/R;
eq5:(ve-vf)/R+(ve-va)/R+(ve-vh)/R;
eq6:(vf-ve)/R+(vf-vg)/R+(vf-vb)/R;
eq7:(vg-vf)/R+(vg-vh)/R+(vg-vc)/R;
eq8:(vh-vd)/R+(vh-ve)/R+(vh-vg)/R;
eq9:solve([eq2,eq3,eq4,eq5,eq6,eq7],[vb,vc,vd,ve,vf,vg]),va=1,vh=0;
eq10:eq9[1];
vb:eq10[1];
vd:eq10[3];
ve:eq10[4];
va:1;
Iae:(va-ve)/R,ve;
Iad:(va-vd)/R,vd;
Iab:(va-vb)/R,vb;
Iin:Iae + Iad + Iab;
Req:1/Iin;
```

Maxima Results for Example 1, all R's = R:

```
(%i17) eq2:(vb-va)/R+(vb-vf)/R+(vb-vc)/R;
       eq3:(vc-vb)/R+(vc-vg)/R+(vc-vd)/R;
       eq4:(vd-va)/R+(vd-vh)/R+(vd-vc)/R;
       eq5:(ve-vf)/R+(ve-va)/R+(ve-vh)/R;
       eq6:(vf-ve)/R+(vf-vg)/R+(vf-vb)/R;
       eq7:(vg-vf)/R+(vg-vh)/R+(vg-vc)/R;
       eq9:solve([eq2,eq3,eq4,eq5,eq6,eq7],[vb,vc,vd,ve,vf,vg]),va=1,vh=0;
       eq10:eq9[1];
       vb:eq10[1];
       vd:eq10[3];
       ve:eq10[4];
       va:1;
       Iae:(va-ve)/R,ve;
       Iad:(va-vd)/R,vd;
       Iab:(va-vb)/R,vb;
       Iin:Iae + Iad + Iab;
       Req:1/Iin;
```

(eq2) $\frac{vb-vf}{R}+\frac{vb-vc}{R}+\frac{vb-va}{R}$

(eq3) $\frac{vc-vg}{R}+\frac{vc-vd}{R}+\frac{vc-vb}{R}$

(eq4) $\frac{vd-vh}{R}+\frac{vd-vc}{R}+\frac{vd-va}{R}$

(eq5) $\frac{ve-vh}{R}+\frac{ve-vf}{R}+\frac{ve-va}{R}$

(eq6) $\frac{vf-vg}{R}+\frac{vf-ve}{R}+\frac{vf-vb}{R}$

(eq7) $\frac{vg-vh}{R}+\frac{vg-vf}{R}+\frac{vg-vc}{R}$

(eq9) $[[vb=\frac{2}{3},vc=\frac{1}{2},vd=\frac{1}{2},ve=\frac{1}{2},vf=\frac{1}{2},vg=\frac{1}{3}]]$

(eq10) $[vb=\frac{2}{3},vc=\frac{1}{2},vd=\frac{1}{2},ve=\frac{1}{2},vf=\frac{1}{2},vg=\frac{1}{3}]$

(vb) $vb=\frac{2}{3}$

(vd) $vd=\frac{1}{2}$

(ve) $ve=\frac{1}{2}$

(va) 1

(Iae) $\frac{1}{2R}$

(Iad) $\frac{1}{2R}$

(Iab) $\frac{1}{3R}$

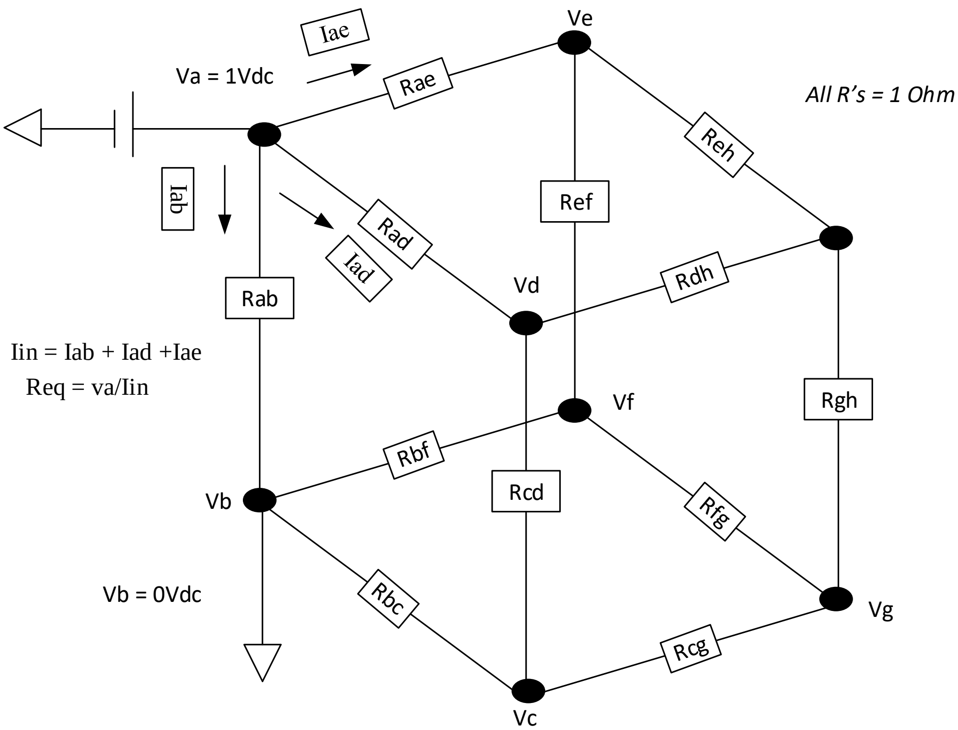
(Iin) $\frac{4}{3R}$

(Req) $\frac{3R}{4}$

The Correct Answer "Req_ah" = 3R/4 is shown.

Let's try example 2 with all "R's" the same:

Example 2
Question:
If all R's = R, determine the equivalent resistance from point a to point b (Req_ab)?
Answer:
Req_ab = 7R/12



Note that in example 2, the Ohmmeter is placed between points "a" and "b". So, "Va" is known and assume it is 1 Vdc. "Vb" is known and assume it is 0 Vdc.

Since "Va" and "Vb" are known, the technique is to remove nodal "equation 1" for "Va" and nodal "equation 2 for "Vb".

Maxima input:

```
eq1:(va-ve)/R+(va-vd)/R+(va-vb)/R;
eq2:(vb-va)/R+(vb-vf)/R+(vb-vc)/R;
eq3:(vc-vb)/R+(vc-vg)/R+(vc-vd)/R;
eq4:(vd-va)/R+(vd-vh)/R+(vd-vc)/R;
eq5:(ve-vf)/R+(ve-va)/R+(ve-vh)/R;
eq6:(vf-ve)/R+(vf-vg)/R+(vf-vb)/R;
eq7:(vg-vf)/R+(vg-vh)/R+(vg-vc)/R;
eq8:(vh-vd)/R+(vh-ve)/R+(vh-vg)/R;
eq9:solve([eq3,eq4,eq5,eq6,eq7,eq8],[vc,vd,ve,vf,vg,vh]),va=1,vb=0;
eq10:eq9[1];
vb:0;
vd:eq10[2];
ve:eq10[3];
va:1;
Iae:(va-ve)/R,ve;
Iad:(va-vd)/R,vd;
Iab:(va-vb)/R;
Iin:Iae + Iad + Iab;
Req:1/Iin;
```

Maxima Results:

```
(%i19) eq1:(va−ve)/R+(va−vd)/R+(va−vb)/R;
       eq2:(vb−va)/R+(vb−vf)/R+(vb−vc)/R;
       eq3:(vc−vb)/R+(vc−vg)/R+(vc−vd)/R;
       eq4:(vd−va)/R+(vd−vh)/R+(vd−vc)/R;
       eq5:(ve−vf)/R+(ve−va)/R+(ve−vh)/R;
       eq6:(vf−ve)/R+(vf−vg)/R+(vf−vb)/R;
       eq7:(vg−vf)/R+(vg−vh)/R+(vg−vc)/R;
       eq8:(vh−vd)/R+(vh−ve)/R+(vh−vg)/R;
       eq9:solve([eq3,eq4,eq5,eq6,eq7,eq8],[vc,vd,ve,vf,vg,vh]),va=1,vb=0;
       eq10:eq9[1];
       vb:0;
       vd:eq10[2];
       ve:eq10[3];
       va:1;
       lae:(va−ve)/R,ve;
       lad:(va−vd)/R,vd;
       lab:(va−vb)/R;
       lin:lae + lad + lab;
       Req:1/lin;
```

(eq1) $\frac{va-ve}{R}+\frac{va-vd}{R}+\frac{va-vb}{R}$

(eq2) $\frac{vb-vf}{R}+\frac{vb-vc}{R}+\frac{vb-va}{R}$

(eq3) $\frac{vc-vg}{R}+\frac{vc-vd}{R}+\frac{vc-vb}{R}$

(eq4) $\frac{vd-vh}{R}+\frac{vd-vc}{R}+\frac{vd-va}{R}$

(eq5) $\frac{ve-vh}{R}+\frac{ve-vf}{R}+\frac{ve-va}{R}$

(eq6) $\frac{vf-vg}{R}+\frac{vf-ve}{R}+\frac{vf-vb}{R}$

(eq7) $\frac{vg-vh}{R}+\frac{vg-vf}{R}+\frac{vg-vc}{R}$

(eq8) $\frac{vh-vg}{R}+\frac{vh-ve}{R}+\frac{vh-vd}{R}$

(eq9) $[[vc=\frac{5}{14},vd=\frac{9}{14},ve=\frac{9}{14},vf=\frac{5}{14},vg=\frac{3}{7},vh=\frac{4}{7}]]$

(eq10) $[vc=\frac{5}{14},vd=\frac{9}{14},ve=\frac{9}{14},vf=\frac{5}{14},vg=\frac{3}{7},vh=\frac{4}{7}]$

(vb) 0

(vd) $vd=\frac{9}{14}$

(ve) $ve=\frac{9}{14}$

(va) 1

(lae) $\frac{5}{14\,R}$

(lad) $\frac{5}{14\,R}$

(lab) $\frac{1}{R}$

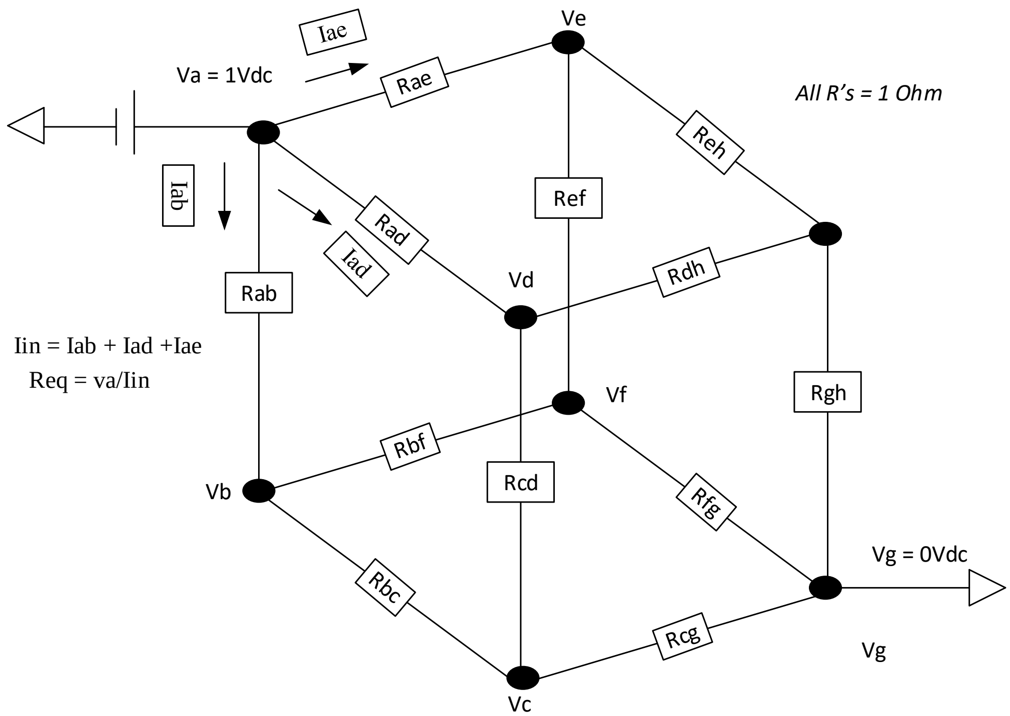
(lin) $\frac{12}{7\,R}$

(Req) $\frac{7\,R}{12}$

The Correct Answer "Req_ab" = 7R/12 is shown.

Let's try example 3 with all "R's" the same:

Example 3
Question:
If all R's = R, determine the equivalent resistance from point a to point g (Req_ag)?
Answer:
Req_ag = 5R/6



Note that in example 3, the Ohmmeter is placed between points "a" and "g".
So, "Va" is known and assume it is 1 Vdc. "Vg" is known and assume it is 0 Vdc.

Since "Va" and "Vg" are known, the technique is to remove nodal "equation 1" for "Va" and nodal "equation 7 for "Vg".

Maxima input:

```
eq1:(va-ve)/R+(va-vd)/R+(va-vb)/R;
eq2:(vb-va)/R+(vb-vf)/R+(vb-vc)/R;
eq3:(vc-vb)/R+(vc-vg)/R+(vc-vd)/R;
eq4:(vd-va)/R+(vd-vh)/R+(vd-vc)/R;
eq5:(ve-vf)/R+(ve-va)/R+(ve-vh)/R;
eq6:(vf-ve)/R+(vf-vg)/R+(vf-vb)/R;
eq7:(vg-vf)/R+(vg-vh)/R+(vg-vc)/R;
eq8:(vh-vd)/R+(vh-ve)/R+(vh-vg)/R;
eq9:solve([eq2,eq3,eq4,eq5,eq6,eq8],[vb,vc,vd,ve,vf,vh]),va=1,vg=0;
eq10:eq9[1];
vb:eq10[1];
vd:eq10[3];
ve:eq10[4];
va:1;
Iae:(va-ve)/R,ve;
Iad:(va-vd)/R,vd;
Iab:(va-vb)/R,vb;
Iin:Iae + Iad + Iab;
Req:1/Iin;
```

Maxima Results for example3 for all R's=R:

```
(%i19) eq1:(va-ve)/R+(va-vd)/R+(va-vb)/R;
       eq2:(vb-va)/R+(vb-vf)/R+(vb-vc)/R;
       eq3:(vc-vb)/R+(vc-vg)/R+(vc-vd)/R;
       eq4:(vd-va)/R+(vd-vh)/R+(vd-vc)/R;
       eq5:(ve-vf)/R+(ve-va)/R+(ve-vh)/R;
       eq6:(vf-ve)/R+(vf-vg)/R+(vf-vb)/R;
       eq7:(vg-vf)/R+(vg-vh)/R+(vg-vc)/R;
       eq8:(vh-vd)/R+(vh-ve)/R+(vh-vg)/R;
       eq9:solve([eq2,eq3,eq4,eq5,eq6,eq8],[vb,vc,vd,ve,vf,vh]),va=1,vg=0;
       eq10:eq9[1];
       vb:eq10[1];
       vd:eq10[3];
       ve:eq10[4];
       va:1;
       lae:(va-ve)/R,ve;
       lad:(va-vd)/R,vd;
       lab:(va-vb)/R,vb;
       lin:lae + lad + lab;
       Req:1/lin;
```

(eq1) $\frac{va-ve}{R}+\frac{va-vd}{R}+\frac{va-vb}{R}$

(eq2) $\frac{vb-vf}{R}+\frac{vb-vc}{R}+\frac{vb-va}{R}$

(eq3) $\frac{vc-vg}{R}+\frac{vc-vd}{R}+\frac{vc-vb}{R}$

(eq4) $\frac{vd-vh}{R}+\frac{vd-vc}{R}+\frac{vd-va}{R}$

(eq5) $\frac{ve-vh}{R}+\frac{ve-vf}{R}+\frac{ve-va}{R}$

(eq6) $\frac{vf-vg}{R}+\frac{vf-ve}{R}+\frac{vf-vb}{R}$

(eq7) $\frac{vg-vh}{R}+\frac{vg-vf}{R}+\frac{vg-vc}{R}$

(eq8) $\frac{vh-vg}{R}+\frac{vh-ve}{R}+\frac{vh-vd}{R}$

(eq9) $[[vb=\frac{3}{5},vc=\frac{2}{5},vd=\frac{3}{5},ve=\frac{3}{5},vf=\frac{2}{5},vh=\frac{2}{5}]]$

(eq10) $[vb=\frac{3}{5},vc=\frac{2}{5},vd=\frac{3}{5},ve=\frac{3}{5},vf=\frac{2}{5},vh=\frac{2}{5}]$

(vb) $vb=\frac{3}{5}$

(vd) $vd=\frac{3}{5}$

(ve) $ve=\frac{3}{5}$

(va) 1

(lae) $\frac{2}{5R}$

(lad) $\frac{2}{5R}$

(lab) $\frac{2}{5R}$

(lin) $\frac{6}{5R}$

(Req) $\frac{5R}{6}$

The Correct Answer "Req_ag" = 5R/6 is shown.