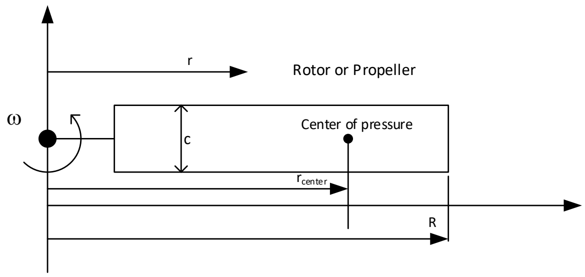
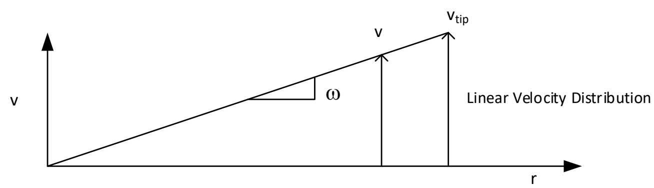
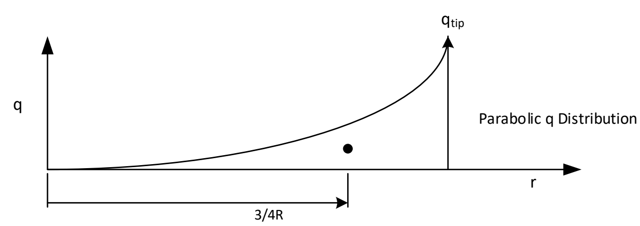
06/25/2022 – Saturday - Propellers and Rotors

Propellers – Fixed Pitch = Airplanes & Quadcopters
Rotors – Variable Pitch = Helicopters

Simplified Analysis - Assume the propellers or rotors in this analysis have no twist and are flat plates with no airfoil.







$$v = \omega\, r$$

$$q = \frac{1}{2}\,\rho\, v^2 = \frac{1}{2}\,\rho\,\omega^2\, r^2$$

$$v_{tip} = \omega\, R$$

$$q_{tip} = \frac{1}{2}\,\rho\,\omega^2\, R^2$$

Find the average dynamic pressure of the rotor:

$$q_{ave} = \frac{1}{R}\int_0^R \frac{1}{2}\,\rho\,\omega^2\, r^2\, dr = \frac{\rho\,\omega^2}{2\,R}\int_0^R r^{\,2} dr = \frac{\rho\,\omega^2}{2\,R}\frac{R^3}{3} = \frac{\rho\,\omega^2 R^2}{6}$$

Centroid Equation:

$$\overline{x} = \frac{\int_a^b x\, y(x) dx}{\int_a^b y(x) dx}$$

Find the location of the center of pressure on the rotor:

$$r_{center} = \frac{\int_0^R \frac{1}{2}\,\rho\,\omega^2\, r^2\, r\, dr}{\int_0^R \frac{1}{2}\,\rho\,\omega^2\, r^2\, dr} = \frac{\frac{1}{2}\,\rho\,\omega^2 \int_0^R r^3\, dr}{\frac{1}{2}\,\rho\,\omega^2 \int_0^R r^2\, dr} = \frac{\frac{R^4}{4}}{\frac{R^3}{3}} = \frac{3}{4}\, R$$

Propeller Example:

We have a two bladed 6” x 3 propeller for a model airplane that is spinning at 10,000 RPM. The chord of the blade is 0.6 inches. Determine the blade pitch. Determine the average dynamic pressure. Determine the thrust the propeller provides, and the power needed to turn the propeller at sea level.

Determine Blade Pitch:

D = 6 inches = Diameter
P = 3 = Pitch Displacement = for every revolution of the propeller, 3 inches of air is displaced.
C = 0.6 inches = Blade Chord
R = D/2 = 3 inches = Radius of the propeller

$$\tan(\theta) = \frac{P}{\pi\, D}$$

$$\theta = \tan^{-1}\left(\frac{3}{\pi\, 6}\right) = 9.043° = 0.1578\, rad$$

The speed of the propeller:

$$\omega = \; 10{,}000\, \frac{rev}{min} \cdot 2\,\pi\, \frac{rad}{rev} \cdot \frac{min}{60\, sec} = 1047.197\, \frac{rad}{sec}$$

Density of air at sea level:

$$\rho = 0.0023769\, \frac{lb\, sec^2}{ft^4} \cdot \left(\frac{ft}{12\, in}\right)^4 = 1.1462x10^{-7} \frac{lb\, sec^2}{in^4}$$

Average Dynamic Pressure:

$$q_{ave} = \frac{\rho\, \omega^2 R^2}{6} = \frac{1.1462x10^{-7} \frac{lb\, sec^2}{in^4} \left(1047.197 \frac{rad}{sec}\right)^2 (3\, in)^2}{6} = 0.1885\, \frac{lb}{in^2}$$

Compute the propeller thrust: (S = surface area of the propeller = R * C)

$$F = 2\, q_{ave}\, S = 2\, \left(0.1885\, \frac{lb}{in^2}\right)(3\, in)(0.6\, in) = 0.6786\, lb$$

Compute the Velocity of the air moving through the propeller:

$$V = 10{,}000\, \frac{rev}{min} \cdot \frac{min}{60\, sec} \cdot \frac{3\, in}{rev} = 500\, \frac{in}{sec}$$

Computer the Power required to drive the propeller:

$$P = F\, V = 0.6786\, lb\, \cdot 500\, \frac{in}{sec} = 339.3\, \frac{in\, lb}{sec}$$

$$1\, hp = 6600\, \frac{in\, lb}{sec} = 746\, Watts$$

$$P = 339.3\, \frac{in\, lb}{sec} \cdot \frac{hp}{6600\, \frac{in\, lb}{sec}} = 0.0514\, hp$$

$$P = 0.0514\, hp\, \cdot \frac{746\, Watts}{hp} = 38.35\, Watts$$

Helicopter Rotor Example:

For a four bladed main rotor, used on a single rotor helicopter that spins at 250 rpm and with each blade 20 feet long with a chord of 2 feet, compute the lift generated as a function of its collective pitch. The helicopter has the weight of 10,000 pounds and is at sea level.

Determine the collective pitch of the rotor required to hover the helicopter. Compute the power required to hover the helicopter.

Density of air at sea level:

$$\rho = 0.0023769 \, \frac{lb \, sec^2}{ft^4}$$

The speed of the rotor:

$$\omega = \; 250 \, \frac{rev}{min} \cdot 2\,\pi \, \frac{rad}{rev} \cdot \frac{min}{60 \, sec} = 26.180 \, \frac{rad}{sec}$$

Coefficient of Lift for a Flat Plate:

$$C_l = 2\pi$$

Coefficient of Drag for an Airfoil:

$$C_{daf} = 0.045$$

Coefficient of Drag for a Flat Plate:

$$C_{dfp} = 1.28$$

Radius of the Rotor:

$$R_r = 20 \, ft$$

Chord of the Rotor:

$$C_r = 2 \, ft$$

Surface Area of the Rotor:

$$S_r = R_r \, C_r = 20 \, ft \; \cdot 2 \, ft = 40 \, ft^2$$

Average dynamic pressure for the Rotor Blade:

$$q_{ave} = \frac{\rho \, \omega^2 \, {R_r}^2}{6} = \frac{0.0023769 \, \frac{lb \, sec^2}{ft^4} \cdot \left(26.180 \, \frac{rad}{sec}\right)^2 \cdot (20 \, ft)^2}{6} = 108.61 \, \frac{lb}{ft^2}$$

Lift for the Rotor Blade:

$$L_r = C_l \, q_{ave} \, S_r \, \theta_r = 2\pi \, \left(108.61 \, \frac{lb}{ft^2}\right) \, (40 \, ft^2) \, \theta_r = 27{,}296.67 \, \frac{lb}{rad} \, \theta_r$$

D1 Drag for the Rotor Blade:

$$D1_r = C_{daf} \, q_{ave} \, S_r = 0.045 \, \left(108.61 \, \frac{lb}{ft^2}\right) (40 \, ft^2) = 195.498 \, lb$$

D2 Drag for the Rotor Blade:

$$D2_r = C_{dfp} \, q_{ave} \, S_r \, \theta_r = 1.28 \, \left(108.61 \, \frac{lb}{ft^2}\right) \, (40 \, ft^2) \, \theta_r = 5{,}560.832 \, \frac{lb}{rad} \, \theta_r$$

Lift for a 4-bladed Rotor Blade as a Function of Collective Pitch?

$$L_{4r} = 4 \, L_r = 4 \cdot 27{,}296.67 \, \frac{lb}{rad} \, \theta_r = \; 109{,}186.68 \, \frac{lb}{rad} \, \theta_r$$

What is the Collective Pitch Required for the Helicopter to lift its own weight?

$$\theta_r = \frac{10{,}000\ lb}{109{,}186.68\ \frac{lb}{rad}} = 0.0916\ rad = 5.247\ deg$$

What is the Drag Force of the Rotor when the Helicopter is hovering?

$$D_{4r} = 4\ (195.498\ lb) + 4\ \left(5{,}560.832\ \frac{lb}{rad} \cdot 0.0916\ rad\right) = 2{,}819.48\ lb$$

What is the Torque on the Rotor when the Helicopter is hovering?

$$\Gamma_{4r} = \frac{3}{4}\ R_r\ D_{4r} = \frac{3}{4} \cdot 20\ ft\ \cdot 2{,}819.48\ lb = 42{,}292.20\ ft\ lb$$

What is the power required for the Rotor when the Helicopter is Hovering?

$$P_r = \Gamma_{4r} \cdot \omega = 42{,}292.20\ ft\ lb\ \cdot 26.180\ \frac{rad}{sec} = 1{,}107{,}209.79\ \frac{ft\ lb}{sec}$$

$$1\ hp = 550\ \frac{ft\ lb}{sec}$$

$$P_r = \frac{1{,}107{,}209.79\ \frac{ft\ lb}{sec}}{550\ \frac{\frac{ft\ lb}{sec}}{hp}} = 2{,}013\ hp$$